

## TWO-PHASE STRUCTURE OF THE CONDENSATION BOUNDARY LAYER WITH A NON-CONDENSING GAS AND LIQUID DROPLETS

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**Abstract**—Condensation heat transfer and the structure of the dispersed, two-phase, two-component boundary layer are studied under forced convection conditions. An approximate analytical relation is found between the profiles of droplet mass fraction and temperature in the boundary layer. In the case of small temperature differences, the droplet mass fraction decreases for  $Le < 1$  (Lewis number) and increases for  $Le > 1$  as the temperature decreases in the condensation boundary layer. For higher temperature differences, the droplet mass fraction grows rapidly near the interface for all values of  $Le$ . The saturation condition imposed all over the boundary layer by previous authors cannot be fulfilled for  $Le < 1$  unless the main stream droplet mass fraction is higher than a certain minimum value determined in this paper. The heat flux transferred to the wall is only slightly affected by droplets being present in the boundary layer.

**Key Words:** condensation, non-condensing gas, droplets, boundary layer, two-phase structure

### INTRODUCTION

Forced convection condensation on a flat plate in the presence of a non-condensing gas has been studied by many authors (e.g. Sparrow *et al.* 1967; Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch 1985, 1986). The main goal of these papers has been to calculate the mass and heat fluxes transferred to the plate. The laminar boundary layer equations have been solved in different ways and sometimes contradictory assumptions have been employed.

In the work by Sparrow *et al.* (1967) it was assumed that the vapour was saturated in the main stream and on the interface between the gas phase and liquid film. No other assumptions have been made concerning the thermodynamic state of the vapour in the boundary layer. In particular, the possibility of droplet formation has not been considered. Furthermore, the thin film approximation has been made based on the results of Cess (1960) and Koh (1962).

Another approach was adopted in the paper by Hijikata & Mori (1973). They assumed that the vapour is saturated throughout the boundary layer and that droplets appear to avoid any metastable state of vapour. The appearance of droplets in the boundary layer results in a considerable complication of the model. So, to facilitate the main task of the heat flux calculation, the authors solved two-phase boundary layer equations by means of the integral method. Moreover, they neglected the heat resistance of the film, assuming that the temperature of the interface between the liquid film and dispersed two-phase medium was equal to the temperature of the plate surface. The same assumptions have been adopted by Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986). However, they numerically solved the two-phase boundary layer equations by a finite difference method.

Still another approach to the thermodynamic modelling of the boundary layer has been proposed by Poinot & Huetz (1985) and Sekulić (1985) for somewhat different problems. Poinot & Huetz (1985) studied the condensation of quiescent vapour in the presence of non-condensing gas on a cylindrical surface and Sekulić (1985) investigated the problem of free convection condensation in

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the presence of non-condensing gas on a vertical cryosurface. In both papers, the so-called critical supersaturation model of homogeneous nucleation (Abraham, 1974) was applied for the problem of fog formation. However, the experimental observations of Sekulić (1985) showed that droplets had formed at some supersaturation much less than the critical supersaturation. This has been attributed to heterogeneous nucleation in the boundary layer. Thus, the critical supersaturation model for fog formation in the boundary layer may be considered an extreme case which is applicable only for substances of extremely high purity. However, for most practical applications, it is quite reasonable to assume that droplets appear at the supersaturation line (i.e. at supersaturation = 1). Thus, the critical supersaturation model will not be considered in this paper.

The single-phase model (Sparrow *et al.* 1967) and the critical supersaturation model (Poinsoot & Huetz 1985; Sekulić 1985) represent two extreme cases for the physical picture of the condensation boundary layer. It would seem that the physical assumption adopted in Hijikata & Mori (1973), Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986) (i.e. vapour saturation over the entire boundary layer and droplet formation to avoid any metastable state of vapour) was the best one for the problems of forced convection condensation in the presence of non-condensing gas. However, Hijikata & Mori (1973) pointed out that, in the case of  $Le < 1$  (Lewis number) and for low temperature differences, a condensing vapour is in a superheated state in the single-phase boundary layer. However, they did not show any numerical results to support such a statement. Further, no attention has been given to this problem in Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986), where no numerical values of droplet mass fraction were presented. At the same time, droplets were not observed in the experiments with a steam-air mixture (Legay-Desesquelles 1984) unless the temperature difference was  $> 20$  K. Clement (1985) investigated an aerosol growth or evaporation induced by heat and mass transfer in the presence of non-condensing gas. According to a qualitative analysis of balance equations he concluded that in the case of wall cooling the aerosol evaporates when  $Le < 1$  (e.g. steam-air mixture) and grows when  $Le > 1$ . The qualitative results of Clement (1985) were confirmed by the results of a numerical solution of the one-dimensional, non-stationary problem for a steam-air mixture (Barrett & Clement 1986).

These results compel someone who wishes to deal with forced convection condensation in the presence of non-condensing gas to examine carefully the intrinsic consistency of the models based on the saturation condition throughout the boundary layer (Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch 1985, 1986). Hence, an aim of this paper is to determine the conditions which should be fulfilled in order that vapour be saturated all through the boundary layer. Moreover, it is necessary to determine the validity of the assumption (Hijikata & Mori 1973; Legay-Desesquelles 1984, Legay-Desesquelles & Prunet-Foch 1985, 1986) that the temperature of the interface between the dispersed two-phase medium and the liquid film is equal to the wall temperature. Finally, the introduction of droplets into the modelling of the condensation boundary layer results in a considerable complication of the problem and it is interesting to determine the influence of droplets on the heat flux transferred to the wall.

In the following sections equations describing the homogeneous, two-phase, two-component boundary layer are presented. Liquid film equations are written for the thin film approximation (Cess 1960; Sparrow *et al.* 1967). Then, balance equations on the interface between the liquid film and the dispersed, two-phase, two-component medium are presented.

The problem has been treated in two different ways. Firstly, an approximate analytical relation between temperature and droplet mass fraction is found. This relation makes it possible to discuss the structure of the two-phase boundary layer with regard to its dependence on  $Le$ . Secondly, the system of equations with appropriate boundary conditions is numerically solved and profiles of droplet mass fraction are shown for the case of steam-air mixtures. Finally, the results of this paper are discussed and conclusions are formulated with special reference to the modelling of condensation boundary layers in the presence of non-condensing gas. This paper is a slightly different and abridged version of our previous report (Matuszkiewicz *et al.* 1989), where more details may be found.

FORMULATION OF THE PROBLEM

A homogeneous fluid mixture, consisting of non-condensing gas, saturated vapour and finely dispersed droplets of the same substance is flowing along a flat plate (figure 1). The uniform temperature of the plate is lower than the temperature of the main flow. So, the vapour condenses on the plate and the liquid film arising on it is dragged by the shear stress. A laminar boundary layer develops in the region of the dispersed, two-phase, two-component medium. Quantities of interest are the heat flux transferred to the wall and the profile of the droplet mass fraction. To solve the problem it is necessary to write down the balance equations for the dispersed, two-phase, two-component medium, the balance equations for the liquid film and the balance equations for the interface between the dispersed, two-phase, two-component medium and the liquid film.

GENERAL EQUATIONS OF A HOMOGENEOUS, TWO-PHASE, TWO-COMPONENT BOUNDARY LAYER

In previous works (Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch 1985, 1986) the dispersed liquid phase was considered as a third component in the vapour-gas-droplets mixture. However, component and phase are different notions. Components are parts of the medium which are mixed at a molecular level, whereas phases are the parts of the medium which are mixed at a macroscopic level. So, vapour and non-condensing gas are components in the gas phase while droplets are one of two phases in the two-phase, two-component medium. Accordingly, two different mass fractions should be defined for the problem at hand. The vapour mass fraction in the gas phase is

$$\omega = \frac{\rho_V}{\rho_G}, \tag{1}$$

where  $\rho$  is density, the index V refers to the vapour and G refers to the gas phase. The droplet mass fraction in the two-phase, two-component medium is

$$\Omega = \frac{\epsilon \rho_L}{\rho}, \tag{2}$$

where  $\rho$  is the density of the two-phase, two-component medium, the index L refers to the liquid phase and  $\epsilon$  is the volume fraction of the dispersed liquid phase. The factor  $\epsilon \rho_L$  is the apparent droplet density in the two-phase, two-component medium.

$$T_\infty > T_w$$

$$\omega = \omega_{SAT}(T)$$

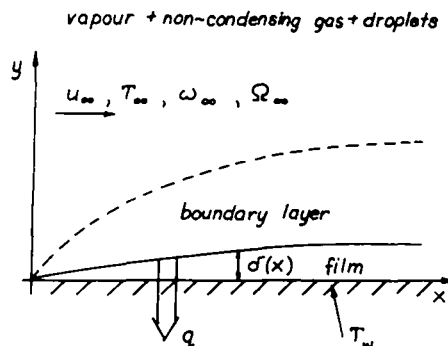


Figure 1. Physical model and coordinate system.

To derive the equations of the homogeneous, two-phase, two-component boundary layer the following assumptions are adopted (Matuszkiewicz *et al.* 1989):

1. The velocities of the gas and dispersed liquid phase are equal.
2. The temperature of the gas and dispersed liquid phase are equal.
3. Linear constitutive laws are valid for the homogeneous, two-phase, two-component medium. Cross phenomena (e.g. thermal diffusion) are negligible.
4. Single-phase boundary layer assumptions are acceptable for the homogeneous, two-phase, two-component flow.
5. Non-condensing gas does not dissolve in the liquid phase.
6. The surface tension effect on the vapour saturation pressure is neglected.
7. Vapour is saturated throughout the boundary layer.
8. Pressure is constant in the boundary layer.

Assumptions 1 and 2 are inherent in the so-called homogeneous, two-phase flow model for a single-component medium and they may be adopted here for the two-component medium without introducing any further simplifications into the model. Assumption 3 indicates that the transport coefficients (i.e. dynamic viscosity and thermal conductivity) should be effective quantities for the homogeneous, two-phase, two-component medium. The transport coefficients of the gas phase (two-component mixture) are calculated from the semiempirical formula of Wilke (cf. Bird *et al.* 1960) and the effective transport coefficients of the homogeneous, two-phase medium are calculated from the formula of Taylor (1971). Assumption 4 will be valid if the droplet diameters are much less than the boundary layer thickness: for a particular choice of main flow parameters that defines an upper limit of droplet diameter [ $d \ll (v_x/u_x)^{1/2}$ ]. However, there is always a region near the leading edge where droplet diameter is of the same order of magnitude as or greater than the boundary layer thickness. Then, the model cannot be applied to this region. The consequences and validity of assumption 7 will be discussed later.

Based on assumptions 1–8, the following equations of the homogeneous, two-phase, two-component boundary layer have been derived (Matuszkiewicz *et al.* 1989):

*continuity,*

$$\frac{d\rho}{dt} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0; \quad [3]$$

*momentum,*

$$\rho \frac{du}{dt} = \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right); \quad [4]$$

*diffusion,*

$$\rho(1 - \Omega) \frac{d\omega}{dt} = \frac{\partial}{\partial y} \left[ \rho(1 - \Omega) D \frac{\partial \omega}{\partial y} \right] - \rho(1 - \omega) \frac{d\Omega}{dt}; \quad [5]$$

and

*thermal energy,*

$$\rho c \frac{dT}{dt} = \rho L \frac{d\Omega}{dt} + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right); \quad [6]$$

where the substantial derivative reads

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}. \quad [7]$$

The meaning of the symbols in [3]–[7] is as follows:  $t$  is the time;  $x, y$  are the Cartesian coordinates;  $u$  is the velocity component parallel to the wall;  $v$  is the velocity component perpendicular to the wall;  $\mu$  is the dynamic viscosity;  $D$  is the binary diffusivity;  $\lambda$  is the thermal conductivity;  $c$  is the

heat capacity;  $L$  is the latent heat of evaporation; and  $T$  is the temperature. It is obvious that the following condition has to be fulfilled throughout the boundary layer:

$$\Omega \geq 0. \quad [8]$$

Equations [3] and [4] have the same form as the analogous equations of a single-phase boundary layer. Equations [5] and [6] differ from the analogous equations of a single-phase boundary layer in having source terms proportional to the substantial derivative of the droplet mass fraction. These source terms are due to exchange of mass and energy between the gas and dispersed liquid phase. The source term in the diffusion equation is small unless the droplet mass fraction is of the same order as the vapour mass fraction. In most cases the droplet mass fraction is many times less than the vapour mass fraction, so the source term in the diffusion equation will be neglected hereafter in this paper. Although this restricts the generality of the results, this simplification decouples the thermal and diffusion equations.

### EQUATIONS OF THE LIQUID FILM

The thin film approximation (Cess 1960; Sparrow *et al.* 1967) is made to simplify the conservation equations of the liquid film. This approximation consists of neglecting inertia terms in the momentum equation and convective terms in the thermal energy equation. It has been proved (Koh 1962) that this yields very good results in the case of forced convection condensation of a pure vapour. The stationary equations of the liquid film take the following form (Sparrow *et al.* 1967):

*continuity,*

$$\frac{\partial u_F}{\partial x} + \frac{\partial v_F}{\partial y} = 0; \quad [9]$$

*momentum,*

$$\frac{\partial^2 u_F}{\partial y^2} = 0; \quad [10]$$

and

*thermal energy,*

$$\frac{\partial^2 T_F}{\partial y^2} = 0; \quad [11]$$

where the index F refers to the film. Thus, it is clear that the thin film approximation involves linear velocity and temperature profiles.

### INTERFACE BALANCE EQUATIONS

The equations of the homogeneous, two-phase, two-component boundary layer and the equations of the liquid film should be supplemented by interface balance equations. In the boundary layer approximation and for a slightly curved film surface they take the following form (Matuszkiewicz *et al.* 1989):

*total mass,*

$$\rho \left( u \frac{d\delta}{dx} - v \right) = \rho_L \left( u_F \frac{d\delta}{dx} - v_F \right); \quad [12]$$

*momentum,*

$$\mu \frac{\partial u}{\partial y} = \mu_L \frac{\partial u_F}{\partial y}; \quad [13]$$

mass of non-condensing gas,

$$u \frac{d\delta}{dx} - v = \frac{1}{1-\omega} D \frac{\partial \omega}{\partial y}; \quad [14]$$

and

thermal energy,

$$(1-\Omega)\rho \left( u \frac{d\delta}{dx} - v \right) L(T_i) + \lambda \frac{\partial T}{\partial y} - \lambda_L \frac{\partial T_F}{\partial y} = 0; \quad [15]$$

where  $\delta$  is the film thickness and the index  $i$  refers to the interface. Equation [14] is derived from assumption 5 (that the non-condensing gas does not dissolve in the liquid phase). Equations [12]–[14] have the same form as the corresponding equations for the interface between the liquid film and the single-phase, two-component boundary layer (Sparrow *et al.* 1967). However, the dynamic viscosity  $\mu$  in [13] and the thermal conductivity  $\lambda$  in [15] are effective, two-phase, two-component quantities. The thermal energy equation [15] differs from its single-phase counterpart (Sparrow *et al.* 1967) in involving the factor  $(1-\Omega)$  in the first term. This factor is induced by the fact that droplets impacting on the film surface do not release the latent heat of condensation. The interface balance equations need to be completed with jump conditions, i.e. formulae relating fluxes with discontinuities of field variables across the interface. They may be inferred from the irreversible thermodynamics of interfaces. They include some phenomenological coefficients which should be determined by experiments or the molecular theory of matter. Up to now, both methods having failed to give credible results, therefore continuity conditions (zero-jumps) are adopted in this paper.

Two continuity conditions are necessary for the problem at hand. The continuity of the tangential component of velocity and the continuity of temperature. In the boundary layer approximation and for a slightly curved film surface, the continuity of the tangential component of velocity is expressed as follows:

$$u = u_F. \quad [16]$$

The continuity of temperature is

$$T = T_F. \quad [17]$$

It can be proved that the continuity conditions are equivalent to the assumption that thermodynamic processes are reversible (Delhaye 1981).

#### RELATION BETWEEN TEMPERATURE AND DROPLET MASS FRACTION

For saturated vapour, the vapour mass fraction and the temperature are not independent quantities. They are related in the following way:

$$\omega(T) = \frac{mp_v(T)}{1 - \frac{(1-m)p_v(T)}{p}}, \quad [18]$$

where  $p_v = p_v(T)$  is the vapour saturation pressure and  $m = M_v/M_A$  is the ratio of the molar masses of the vapour and non-condensing gas. The saturation assumption throughout the boundary layer allows us to reduce the number of dependent field variables by one. Previously (Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch 1985, 1986), this point has not received enough attention, though [18] was used by these authors. If a profile of the vapour mass fraction is known, then the temperature profile is known and thermal energy equation [6] is a first-order differential equation for the droplet mass fraction.

In the general case it is not easy to find an analytical solution for the droplet mass fraction. However, in the special case of small temperature differences ( $T_x - T_w$ ), based on [5] and [6], it is possible to express the droplet mass fraction as a function of temperature. First, if  $\Omega \ll \omega$ , then

the source term in the diffusion equation [5] may be dropped. Then, all terms including gradients of the transport coefficients are neglected in the diffusion equation [5] and thermal energy equation [6]. The vapour mass fraction being a function of temperature, it is possible to write the following formula:

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\omega_1} \frac{\partial^2 \omega}{\partial y^2} - \frac{\omega_2}{\omega_1^3} \left( \frac{\partial \omega}{\partial y} \right)^2, \quad [19]$$

where  $\omega_1$  and  $\omega_2$  are the first and second temperature derivatives of the vapour mass fraction, respectively. The last term on the r.h.s. of [19] is quadratic in the gradient of vapour mass fraction, so it may be neglected in the case of a small temperature difference. The first term on the r.h.s. of [19] is replaced by an expression resulting from diffusion equation [5] and then [19] may be transformed to the following form:

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{D\omega_1} \frac{d\omega}{dt}. \quad [20]$$

Now, inserting [20] into thermal energy equation [6] and returning to temperature as the main field variable, the following differential equation is obtained:

$$(1 - Le) \frac{dT}{dt} = \left( \frac{c}{L} \right) \frac{d\Omega}{dt}. \quad [21]$$

Integration of [21] yields the following relation between droplet mass fraction and temperature:

$$\Omega - \Omega_\infty = (1 - Le) \left( \frac{L}{c} \right) (T - T_\infty). \quad [22]$$

The temperature difference ( $T - T_\infty$ ) is always negative in the condensation boundary layer. Thus, for:

- (1)  $Le < 1$  (e.g. a steam-air mixture), the droplet mass fraction decreases as the interface is approached;
- (2)  $Le > 1$  (e.g. a mixture of air and ethyl alcohol vapour), the droplet mass fraction increases as the interface is approached;
- (3)  $Le = 1$ , the droplet mass fraction is constant.

If there are no droplets in the main flow, i.e.  $\Omega_\infty = 0$ , it is easily seen from [22] that droplets can be formed in the boundary layer only for  $Le > 1$ . In the case of a steam-air mixture ( $Le < 1$ ), the case treated by Hijikata & Mori (1973), Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986), the droplet mass fraction cannot be negative as the interface is approached. Thus, steam is superheated and assumption 7 is no longer valid. The superheating is due to the fact that the saturation temperature is controlled by the diffusion and that the ratio of the thermal to diffusion boundary layer thickness is proportional to  $(Le)^{1/2}$ .

The results obtained from [22] are compatible with the prediction of Clement (1985) and Barrett & Clement (1986), where aerosol grows in the case of  $Le > 1$  and aerosol evaporates in the case of  $Le < 1$ .

It should be recalled that [22] has been derived by neglecting the non-linear term in the vapour mass fraction gradient in [19], which is true only for small temperature differences. In the general case this non-linear term should be included. The detailed analysis of the thermal energy equation [6] shows that this term is predominant for high temperature differences (Matuszkiewicz *et al.* 1989). In the case of  $Le < 1$ , this term being of opposite sign to the sign of other terms in the thermal energy equation, it is responsible for raising the droplet mass fraction to positive values near the interface for  $\Omega_\infty = 0$  (cf. figures 4-6). However, the vapour is always superheated in the external part of the boundary layer. Hence, for  $Le < 1$  and  $\Omega_\infty = 0$ , the saturation condition adopted in Hijikata & Mori (1973), Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986) is not to be fulfilled over all the condensation boundary layer.

If vapour is not saturated, [18] can still be used to relate the vapour mass fraction to the vapour partial pressure. However, the vapour mass fraction and temperature are independent variables in this case and it is not possible to eliminate one of them from the diffusion equation and thermal

energy equation. The problem of condensation in the single-phase boundary layer, when  $\omega$  and  $T$  are independent variables, has been solved by Minkowycz & Sparrow (1969).

### NUMERICAL TREATMENT

The equations of the condensation boundary layer must be supplemented by appropriate boundary conditions. In the past, two kinds of conditions to be fixed in the main flow have been used. The first consisted of fixing the vapour mass fraction  $\omega_\infty$  and the temperature  $T_\infty$ : then the total pressure was implicitly expressed by [18], with the vapour saturation pressure corresponding to temperature  $T_\infty$  (Sparrow *et al.* 1967). The second consisted of fixing the total pressure and the temperature, the vapour mass fraction being explicitly calculated from [18] (Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch, 1985, 1986). Herein, the former conditions are imposed, thus the main flow conditions are as follows:

$$u = u_\infty, \quad T = T_\infty, \quad \omega = \omega_\infty, \quad \Omega = \Omega_\infty. \quad [23]$$

The boundary conditions at the wall are fixed in the following way:

$$u_F = 0, \quad v_F = 0, \quad T_F = T_w. \quad [24]$$

Hereafter, only stationary processes will be considered. If the region near the leading edge is excluded, a self-similar transformation may be applied to the system of condensation boundary layer equations. It reduces the system to a system of ordinary differential equations. The transformation is defined as follows:

*for the dispersed, two-phase, two-component boundary layer,*

$$\xi = (y - \delta) \left[ \frac{u_\infty}{(v_\infty x)} \right]^{1/2},$$

$$\Psi = (u_\infty v_\infty x)^{1/2} F(\xi); \quad [25]$$

and

*for the liquid film,*

$$\eta = \left[ \frac{u_\infty}{(v_L x)} \right]^{1/2},$$

$$\Psi_F = (u_\infty v_L x)^{1/2} f(\eta); \quad [26]$$

where  $\nu$  is the kinematic viscosity. Transformation [25, 26] has been successfully applied to a single-phase condensation boundary layer by Sparrow *et al.* (1967) and it will prove to be efficient for a dispersed, two-phase, two-component boundary layer. Application of [25] to [3]–[6] leads to the following ordinary differential equations for a dispersed, two-phase, two-component boundary layer:

$$F''' + \frac{1}{2} FF'' = 0, \quad [27]$$

$$\Phi'' + \frac{1}{2} Sc F \Phi' = 0 \quad [28]$$

and

$$\left( \frac{L}{c} \right) \Omega' = (1 - Le) \frac{\omega_\infty - \omega_1}{\omega_1} \Phi' - \left[ \frac{2\omega_2(\omega_\infty - \omega_1)^2}{Pr \omega_1^3} \right] \frac{\Phi'^2}{F}, \quad [29]$$

where  $Sc$  is the Schmidt number,  $Pr$  is the Prandtl number, ' denotes the  $\xi$  derivative and

$$\Phi = \frac{\omega - \omega_1}{\omega_\infty - \omega_1}. \quad [30]$$



The liquid film equations [9]–[11] are easily solved with the aid of [26] in terms of  $\eta, f(\eta)$  variables in the following way:

$$f(\eta) = \frac{1}{2} f''_w(0) \eta^2 \tag{31}$$

and

$$\frac{T - T_w}{T_i - T_w} = \frac{\eta}{\eta_\delta}, \tag{32}$$

where  $f''_w(0)$  is the second derivative of  $f$  on the wall and  $\eta_\delta$  is the value of  $\eta$  for  $y = \delta$ .

To transform the interface balance equations [12]–[15] and continuity conditions [16] by means of [25] and [26] the following steps are taken. Transformation of the mass balance equation [12] with the aid of [31] yields the following differential equation for film thickness:

$$\delta \frac{d\delta}{dx} - \frac{1}{4} \frac{\delta^2}{x} = \frac{1}{2} \frac{\rho}{\rho_L} \frac{(v_{vL})^{1/2}}{u_x} \frac{F(0)}{f''_w(0)}. \tag{33}$$

It is easily solved to result in

$$\delta(x) = \left[ 2 \frac{\rho}{\rho_L} \frac{(v_{vL})^{1/2}}{u_x} \frac{F(0)}{f''_w(0)} x \right]^{1/2}. \tag{34}$$

The interface momentum balance equation is transformed to the following form:

$$R f''_w(0) = F''(0), \tag{35}$$

where

$$R = \frac{\rho_L \mu_L}{\rho \mu}.$$

Equation [35] will be used to eliminate  $f''_w(0)$  from [34]. The following equation is then obtained:

$$\delta(x) = \left[ 2 \frac{F(0)}{F''(0)} \frac{v_L x}{u_x} \right]^{1/2}. \tag{36}$$

The film thickness is expressed in terms of the interfacial parameters of the dispersed, two-phase, two-component boundary layer,  $F(0)$  and  $F''(0)$ . The interfacial balance equations of mass of the non-condensing gas [14] and of thermal energy [15] are transformed to the following forms:

*mass of non-condensing gas,*

$$\frac{1}{2} Sc F(0) = \frac{\omega_\infty - \omega_i}{1 - \omega_i} \Phi'(0); \tag{37}$$

and

*thermal energy,*

$$\frac{1}{2} F(0) \left[ 1 - \Omega_i + \frac{\lambda(1 - \omega_i)}{\rho DL(T_i) \omega_i(T_i)} \right] = \frac{\lambda_L(T_i - T_w)}{\mu_L L(T_i)} \sqrt{R} \frac{1}{\eta_\delta}. \tag{38}$$

where

$$\eta_\delta = \left[ \frac{2F(0)}{F''(0)} \right]^{1/2}.$$

Continuity condition [16] takes the following form:

$$F'(0) = \left[ \frac{2F(0)F''(0)}{R} \right]^{1/2}. \tag{39}$$

In Sparrow *et al.* (1967) it was assumed that  $F'(0) = 0$ . Usually this assumption is acceptable. However, in the case (e.g. high pressure) where  $R$  is not very high it can yield incorrect results.

Finally, the main flow conditions [23], together with the definition [30] of  $\Phi$ , result in the following conditions for variables  $F$ ,  $\Phi$  and  $\Omega$ :

$$F'(\infty) = 1, \quad \Phi(0) = 0, \quad \Phi(\infty) = 1, \quad \Omega(\infty) = \Omega_x. \quad [40]$$

The thin film approximation ([9]–[11]) involved linear profiles of velocity and temperature, and consequently it enabled us to eliminate the film variables. Thus, the problem is reduced to solving only the equations of the dispersed, two-phase, two-component boundary layer [27]–[29], supplemented by non-linear boundary conditions [37]–[40] and by [18], which relates the temperature to the vapour mass fraction on the saturation line. The sixth-order system of ordinary differential equations [27]–[29] was solved by means of a fourth-order Runge–Kutta method. A shooting method was chosen to convert the boundary value problem to an initial value problem, which was iteratively solved. The model equations in this paper and the ones in our previous report (Matuszkiewicz *et al.* 1989) are slightly different but the results we obtained here are generally not different.

### RESULTS AND DISCUSSION

By way of example, calculations were made for a steam–air mixture ( $Le < 1$ ): they were done for one value of the main stream temperature  $T_x = 373$  K and three values of the vapour mass fraction  $\omega_x = 0.3, 0.6$  and  $0.9$ . One value of the main stream droplet mass fraction has been chosen, i.e.  $\Omega_x = 0$ , as was done by Hijikata & Mori (1973), Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986). The range of temperature differences ( $T_x - T_w$ ) was between 5 and 30 K.

First of all, the assumption  $T_i = T_w$ , adopted by Hijikata & Mori (1973), Legay-Desesquelles (1984) and Legay-Desesquelles & Prunet-Foch (1985, 1986), was checked. Figure 2 shows the graph of the non-dimensional temperature difference between the interface and the wall as a function of the overall temperature difference ( $T_x - T_w$ ), for three values of the main stream vapour mass fraction. The temperature difference between the interface and the wall does not depend strongly on the overall temperature difference but it does depend strongly on the main stream vapour mass fraction. It is to be concluded that the assumption  $T_i = T_w$  is not acceptable unless the main stream vapour mass fraction is very low, e.g.  $< 0.1$  for the range of parameters examined herein.

The heat flux transferred to the wall is shown in figure 3. The results of calculations are expressed in terms of the quantity  $Nu/(Re)^{1.2}$  vs the temperature difference ( $T_x - T_w$ ), where Nusselt ( $Nu$ ) and Reynolds numbers ( $Re$ ) are defined in the following way:

$$Nu = \frac{|q|x}{\lambda_L(T_x - T_w)} \quad [41]$$

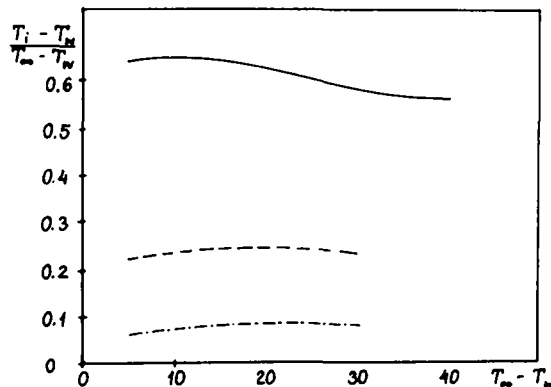


Figure 2. Variation of  $(T_i - T_w)/(T_x - T_w)$  with  $(T_x - T_w)$  for a steam–air system;  $T_x = 373$  K,  $\Omega_x = 0$ :  $\omega_x = 0.9$  (—),  $\omega_x = 0.6$  (---),  $\omega_x = 0.3$  (-·-·-·-).

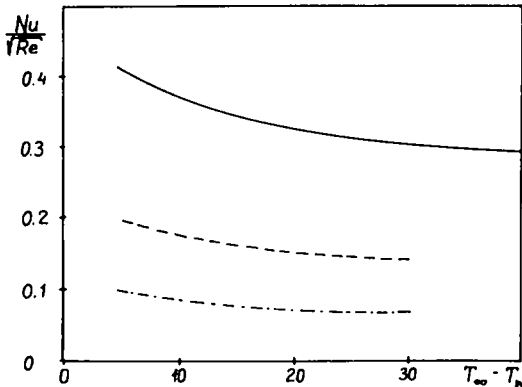


Figure 3. Variation of condensation heat transfer with temperature difference for a steam-air system;  $T_i = 373$  K,  $\Omega_x = 0$ ;  $\omega_x = 0.9$  (—),  $\omega_x = 0.6$  (---),  $\omega_x = 0.3$  (- · - · - ·).

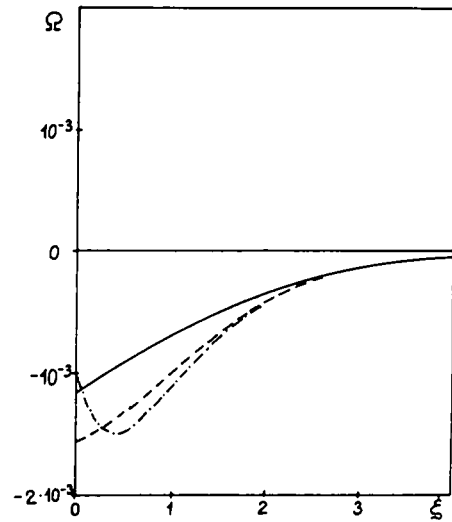


Figure 4. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system ( $Le < 1$ );  $T_i = 373$  K,  $\omega_x = 0.9$ ;  $\Delta T = 10$  K (—),  $\Delta T = 20$  K (---),  $\Delta T = 30$  K (- · - · - ·).

and

$$Re = \frac{u_x x}{\nu_L}, \tag{42}$$

where  $q$  is the heat flux.

When the system of differential equations [27]–[29] is solved, the quantity  $Nu/(Re)^{1/2}$  is easily calculated from the following formula:

$$\frac{Nu}{\sqrt{Re}} = \left[ \frac{1}{2} \frac{F''(0)}{F(0)} \right]^{1/2} \frac{T_i - T_w}{T_x - T_w}. \tag{43}$$

The results reveal that the condensation heat flux is reduced as the vapour mass fraction decreases (figure 3), a phenomenon which has been known for a long time and calculated for a single-phase boundary layer by Sparrow *et al.* (1967).

The most striking results of this paper concern the profiles of droplet mass fraction. As shown above, the droplet mass fraction decreases as the interface is approached in the case of a steam-air mixture ( $Le < 1$ ). Those results are valid for small temperature differences. So, if  $\Omega_x = 0$  is imposed for solving the problem ([27]–[29]), as has been done by previous authors (Hijikata & Mori 1973; Legay-Desesquelles 1984; Legay-Desesquelles & Prunet-Foch, 1985, 1986), negative values of droplet mass fraction are expected. The results of the calculations shown in figures 4–6 reveal this strange feature. Even for 30 K temperature differences the droplet mass fractions are negative in the external part of the boundary layer. Of course, negative values of droplet mass fraction are physically meaningless. However, it may be concluded that vapour is superheated in the region where the droplet mass fraction was found to be negative. The negative values of droplet mass fraction may be considered as a qualitative measure of vapour superheat. For 10 K temperature differences, vapour is superheated all through the boundary layer. For 20 and 30 K temperature differences, vapour is superheated in the external part of the boundary layer and droplets are actually present near the interface (with the exception of  $\omega_x = 0.9$ ). These numerical results are qualitatively confirmed by the experiments of Legay-Desesquelles (1984), where for  $\omega_x < 0.6$  droplets were observed only for temperature differences  $> 20$  K. Hence, the saturation condition is not to be imposed throughout the boundary layer for  $Le < 1$  when there are no droplets in the main flow, i.e.  $\Omega_x = 0$ . Formula [22], confirmed by numerical results, enables us to determine a simple approximate criterion for the saturation condition to be applicable everywhere. It is obvious that restriction [8] should be satisfied all through the boundary layer. For  $Le \geq 1$  it is satisfied for all values of the main stream droplet mass fraction. For  $Le < 1$  it is satisfied only for

$$\Omega_x > \Omega_x^{\min}, \tag{44}$$

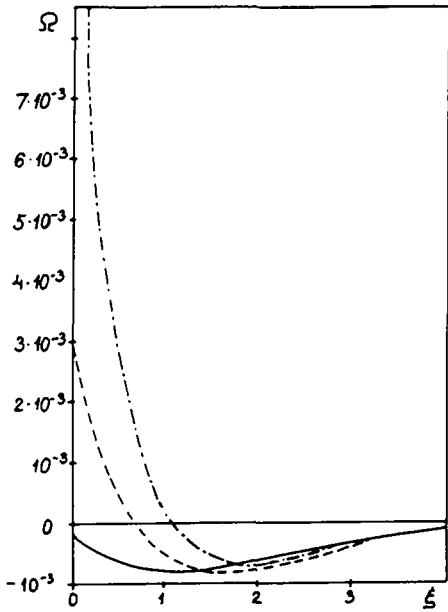


Figure 5. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system ( $Le < 1$ );  $T_x = 373$  K,  $\omega_x = 0.6$ :  $\Delta T = 10$  K (—),  $\Delta T = 20$  K (---),  $\Delta T = 30$  K (-·-·-·).

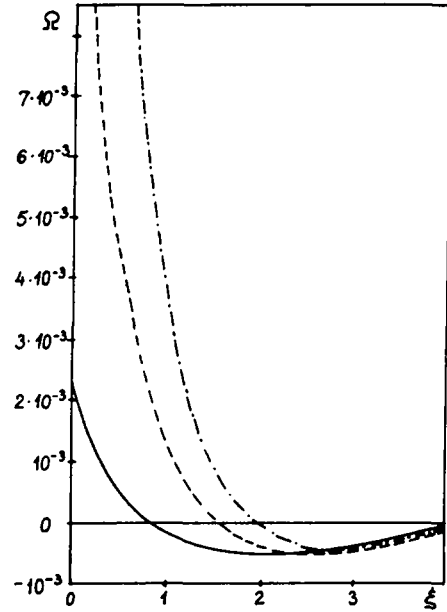


Figure 6. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system ( $Le < 1$ );  $T_x = 373$  K,  $\omega_x = 0.3$ :  $\Delta T = 10$  K (—),  $\Delta T = 20$  K (---),  $\Delta T = 30$  K (-·-·-·).

where  $\Omega_\infty^{\min}$  is a certain minimum value of the main stream droplet mass fraction. This value is easily determined from [22] and [8] in the following way:

$$\Omega_\infty^{\min} = (1 - Le) \frac{c(T_\infty - T_i)}{L(T_\infty)}. \quad [45]$$

For  $Le < 1$  and  $\Omega_\infty < \Omega_\infty^{\min}$  the saturation condition is not to be imposed throughout the condensation boundary layer.

If condition [44] is not fulfilled for  $Le < 1$ , four different cases are possible:

1. The vapour is superheated all through the condensation boundary layer (for  $\Omega_x = 0$ , e.g. figure 4).
2. The condensation boundary layer splits into an external, single-phase sublayer and an internal, two-phase sublayer (for  $\Omega_\infty = 0$ , e.g. figure 6).
3. The condensation boundary layer splits into an external, two-phase sublayer and an internal, single-phase sublayer (for  $0 < \Omega_\infty < \Omega_\infty^{\min}$  and low  $\Delta T$ ).
4. The condensation boundary layer splits into three sublayers—an external two-phase sublayer, a central single-phase sublayer and an internal two-phase sublayer (for  $0 < \Omega_\infty < \Omega_\infty^{\min}$  and high  $\Delta T$ ).

To deal with one of these cases the set of equations [3]–[8], which is valid for a two-phase sublayer, has to be supplemented by a different set of equations valid for a single-phase sublayer. This set can be obtained by putting  $\Omega = 0$  in [3]–[7]. Such a set of equations, with  $\omega$  and  $T$  retained as independent variables, has been solved by Minkowycz & Sparrow (1969). So, to obtain correct profiles of the droplet mass fraction and other physical quantities for  $Le < 1$  and  $0 \leq \Omega_\infty < \Omega_\infty^{\min}$  it is necessary to solve a suitable set of equations in each sublayer, with boundary conditions [23] and [24] imposed at the main flow and at the wall. The boundaries between sublayers are not known and they have to be found by checking condition [8] in a two-phase sublayer and condition  $S \leq 1$  ( $S$  is the supersaturation ratio) in a single-phase sublayer. It is an extremely difficult numerical problem that will not be treated in this paper.

## CONCLUSIONS

By considering the forced convection condensation on a flat plate in the presence of non-condensing gas and liquid droplets the following conclusions have been drawn:

1. As the temperature decreases in the condensation boundary layer the droplet mass fraction decreases slightly in an external part of the boundary layer and increases rapidly near the interface for  $Le < 1$ . In this case the saturation condition cannot be fulfilled all through the boundary layer unless the main stream droplet mass fraction has a certain minimum value. For  $Le > 1$  the droplet mass fraction increases everywhere and the saturation condition can be fulfilled throughout the boundary layer, regardless of the value of the main stream droplet mass fraction.
2. The heat flux transferred to the wall is only slightly influenced by droplets present in the condensation boundary layer.
3. The assumption  $T_i = T_w$ , which has been made in a few recent papers, is not acceptable unless the vapour mass fraction is very low, e.g.  $< 0.1$  for a steam-air mixture.

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